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TECHNICAL NOTE 2119

EFFECT OF HUMIDITY ON PERFORMANCE OF TURBOJET ENGINES

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SUMMARY

The effect of humidity on turbojet-engine performance was theoretically and experimentally investigated. Humidity correction factors that may be used on any turbojet engine and that satisfactorily generalize engine performance over a range of specific humidities are developed.

The humidity effect is shown to be very small with the magnitude depending on the basis of comparison. Correction factors were obtained in the theoretical analysis assuming a constant compressor Mach number with varying humidity and an engine-speed correction resulted. A comparison on a constant engine-speed basis therefore produced a different humidity effect than that obtained at a constant compressor Mach number. The maximum variation of any performance parameter at a constant engine speed was that of thrust; experimental results show a decrease of 3.6 percent in this parameter for a variation in humidity from 30 to 210 grains of water per pound of dry air.

INTRODUCTION

The usual generalized performance parameters for turbojet engines are obtained by assuming that the thermodynamic properties of the working fluid are constant. This assumption leads to some variation in performance of engines under widely differing humidities.

The performance of turbojet engines in air of varying humidity has usually been obtained through a detailed cycle analysis of a representative engine, which accounts for variations in the thermodynamic properties of the working fluid with humidity. The ratio of the performance in dry air to that in moist air is then defined as a humidity correction factor. Correction factors derived in the detailed type of analysis are inconvenient in their application and are inherently restricted to a particular engine.

In the investigation reported herein, which was conducted at the NACA Lewis laboratory, correction factors are developed that may be used for precise correlation of data where performance is affected by some of the thermodynamic-property variations due to atmospheric humidity. Humidity correction factors are developed from a direct consideration of the generalized engine performance parameters. In this manner, a set of humidity correction factors are obtained that are approximate but do not involve parameters associated with a particular engine; hence they may be applied to any engine to correct performance for humidity. An additional advantage of this method of analysis consists of the greatly simplified calculations used to obtain humidity corrections.

The humidity correction factors of the detailed method as applied to two different turbojet engines are compared with each other and with those of the approximate method. In addition, the results of an experimental investigation conducted on a turbojet engine in a variable-humidity altitude chamber are presented to verify the theory.

ANALYSIS

In this analysis, the variation of the thermodynamic properties of the fluid are established and the resulting effects on generalized performance parameters are then determined.

The variation of the thermodynamic properties may be determined by use of the following equations (reference 1), using the symbols of appendix A:

$$C_p = \frac{7000\ C_{p,a} + qC_{p,w}}{7000 + q} \tag{1}$$

$$C_v = \frac{7000\ C_{v,a} + qC_{v,w}}{7000 + q} \tag{2}$$

$$\gamma = \frac{7000\ C_{p,a} + qC_{p,w}}{7000\ C_{v,a} + qC_{v,w}} \tag{3}$$

$$R = \frac{7000\ R_a + qR_w}{7000 + q} \tag{4}$$

In figure 1 an example of the variation of C_p, C_v, γ, and R, as computed from the preceding equations using the data of reference 2, is illustrated over a range of specific humidities. The ordinate is presented in percentage variation from standard values for dry air at 59° F. Similar trends in the variation of these properties occur at the higher temperatures that exist in the turbine or exhaust nozzle of a turbojet engine.

The effect of humidity on the usual generalized engine performance parameters may be studied by investigating the true performance parameters (those with the thermodynamic constants included) as determined from the principles of thermodynamics, fluid dynamics, and flow similarity for two fluid-flow systems: one, an engine operating in dry air and the other, the same engine operating in moist air. Reference 3 shows that the Mach number at any point in the fluid-flow system of a jet engine is a function of the flight Mach number, the Reynolds number, and a characteristic total-temperature ratio. In functional notation,

$$\frac{V}{a} = f_1\left(\frac{V_0}{a_0}, \frac{\rho_0 L V_0}{\mu_0}, \frac{T_3}{T_0}\right) \tag{5}$$

where T_3/T_0 is a ratio of the total temperature at a point downstream of the heat addition to the total temperature at a point upstream of the heat addition. If the temperature ratio and the Reynolds number are assumed to be unaffected by the moisture content of the air, the Mach number at any point throughout the engine is a function only of the flight Mach number. Any performance parameter that is a function of the Mach number at any point within the engine is therefore also a function of the flight Mach number, and for equal flight Mach numbers the performance parameters under moist and dry conditions are equal.

The detailed analysis (appendix B) tends to justify the assumption of a constant temperature ratio because, for a fixed compressor-inlet temperature, the tail-pipe temperature varied the least of any of the performance parameters for two different engines (as will be subsequently shown, from 0.3 to 0.5 percent maximum for variations in humidity up to 300 grains water vapor/lb dry air). The error introduced by both assumptions, that of constant temperature ratio and Reynolds number, cannot be evaluated because the exact form of the function in equation (5) is unknown.

The following analysis concerns the derivation of the true performance parameters in terms of some Mach number within the engine. The parameters are then equated for the moist and dry conditions to determine a humidity correction factor.

The conditions imposed in this approximate method of determining humidity correction factors are: 1295

(1) All engine efficiencies, effective flow areas, and flow coefficients remain constant with varying humidity.

(2) The flight Mach number for one humidity condition is equal to that for the other humidity condition.

(3) The Reynolds numbers at corresponding points for each humidity are equal.

(4) The total-temperature ratios at a selected pair of corresponding points for each humidity are equal.

The jet-thrust humidity correction factor may be determined using the equation

$$F = \frac{\rho_5}{g} A_5 V_5^2 \tag{6}$$

Equation (6) may be rewritten as a function of the Mach number at station 5. Thus

$$F = \rho_5 A_5 \gamma_e R_e t_5 M_5^2 \tag{7}$$

and therefore

$$\frac{F}{\delta \gamma_e} = A_5 \frac{p_5}{P_1} p_{st} M_5^2 \tag{8}$$

but

$$\frac{p_5}{P_1} = \frac{p_0}{P_1} \approx f_2 (M_0) \tag{9}$$

Therefore

$$\frac{F}{\delta \gamma_e} \approx f_3 (M_0) \tag{10}$$

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When the humidity correction factor is defined as the ratio of the performance in dry air to that in moist air, for the moist and dry conditions at equal flight Mach numbers

$$\frac{F_n/\delta}{F_m/\delta} \approx \frac{\gamma_{e,n}}{\gamma_{e,m}} \tag{11}$$

It is shown in one-dimensional-flow theory that the air flow at station 1 is given by the expression

$$\frac{W_a\sqrt{T_1}}{P_1}\sqrt{\frac{R_c}{\gamma_c}} = M_1 A_1 \sqrt{g\left(1 + \frac{\gamma_c - 1}{2} M_1^2\right)^{\frac{\gamma_c + 1}{1 - \gamma_c}}} \tag{12}$$

Therefore

$$\frac{W_a\sqrt{\theta}}{\delta}\sqrt{\frac{R_c}{\gamma_c}} \approx (\text{constant})\ \frac{p_{st}}{\sqrt{T_{st}}}\, f_4\ (M_1) \tag{13}$$

and

$$\frac{W_a\sqrt{\theta}}{\delta}\sqrt{\frac{R_c}{\gamma_c}} \approx f_5\ (M_0) \tag{14}$$

Therefore at constant values of M_0 for operation of the engine with moist and dry air

$$\frac{W_{a,n}\sqrt{\theta}/\delta}{W_{a,m}\sqrt{\theta}/\delta} \approx \sqrt{\frac{R_{c,m}\,\gamma_{c,n}}{R_{c,n}\,\gamma_{c,m}}} \tag{15}$$

where $\sqrt{\frac{R_{c,m}\,\gamma_{c,n}}{R_{c,n}\,\gamma_{c,m}}}$ is the air-flow humidity correction factor.

The fuel-flow correction factor may be obtained by writing the equation of heat balance across the combustion chamber; thus,

$$\eta_b Q_L W_f \approx W_a\ c_{p,b}\ T_3\left(1 - \frac{T_2}{T_3}\right) \tag{16}$$

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But from equation (12)

$$W_a \approx \frac{P_1}{\sqrt{T_1}} \sqrt{\frac{\gamma_c}{R_c}} f_6 \ (M_0) \tag{17}$$

therefore

$$\frac{W_f}{T_1} \approx \frac{P_1 c_{p,b}}{\sqrt{T_1}} \sqrt{\frac{\gamma_c}{R_c}} \frac{T_3}{T_1} \frac{1 - \frac{T_2}{T_3}}{Q_L \eta_b} f_6 \ (M_0) \tag{18}$$

The quantities T_3/T_1 and T_2/T_3 are characteristic total-temperature ratios and have been assumed to be independent of the moisture content of the air. Therefore

$$\frac{W_f}{\delta\sqrt{\theta}} \sqrt{\frac{R_c}{\gamma_c}} \frac{1}{c_{p,b}} \approx f_7 \ (M_0) \tag{19}$$

and for the moist and dry conditions at equal flight Mach numbers

$$\frac{W_{f,n}/\delta\sqrt{\theta}}{W_{f,m}/\delta\sqrt{\theta}} \approx \sqrt{\frac{R_{c,m}\ \gamma_{c,n}}{R_{c,n}\ \gamma_{c,m}}} \frac{c_{p,b,n}}{c_{p,b,m}} \tag{20}$$

where $\sqrt{\frac{R_{c,m}\ \gamma_{c,n}}{R_{c,n}\ \gamma_{c,m}}} \frac{c_{p,b,n}}{c_{p,b,m}}$ is the fuel-flow humidity correction factor.

An engine-speed correction is obtained by equating the compressor Mach number for the two conditions:

$$M_{c,n} = M_{c,m}$$

When it is assumed that the total and static temperatures at the compressor inlet are equal,

$$\frac{N_n}{\sqrt{\gamma_{c,n}R_{c,n}T_{1,n}}} = \frac{N_m}{\sqrt{\gamma_{c,m}R_{c,m}T_{1,m}}}$$

Then the engine-speed humidity correction factor is

$$\frac{N_n/\sqrt{\theta}}{N_m/\sqrt{\theta}} = \sqrt{\frac{\gamma_{c,n}\ R_{c,n}}{\gamma_{c,m}\ R_{c,m}}}$$

Although the humidity correction factors are general in application, the values used for the thermodynamic properties will depend on the engine operating temperatures.

These humidity correction factors are collected for convenience as follows:

Jet thrust, $\left(\frac{\gamma_{e,n}}{\gamma_{e,m}}\right)$

Air flow, $\sqrt{\frac{\gamma_{c,n}\ R_{c,m}}{\gamma_{c,m}\ R_{c,n}}}$

Fuel flow, $\sqrt{\frac{\gamma_{c,n}\ R_{c,m}}{\gamma_{c,m}\ R_{c,n}}}\ \frac{c_{p,b,n}}{c_{p,b,m}}$

Engine speed, $\sqrt{\frac{\gamma_{c,n}\ R_{c,n}}{\gamma_{c,m}\ R_{c,m}}}$

APPARATUS AND PROCEDURE

An experimental investigation was conducted on a turbojet engine in a variable-humidity altitude chamber (fig. 2) to compare the measured effect of humidity on performance with the effect as predicted by this theory. A turbojet engine with a centrifugal-type compressor and single-stage axial-flow turbine was mounted in the altitude chamber on a thrust frame connected through a linkage to a balance-pressure diaphragm-type thrust indicator.

The inlet temperature and pressure and the exhaust pressure of the engine were controlled by electric heaters and automatic valves. Steam at a pressure of 125 pounds per square inch was mixed with the incoming air at a point located 40 feet upstream of the engine in order to facilitate the setting of any specific humidity desired. Just ahead of the engine, a Foxboro Dewcel was installed to determine the specific humidity of the air entering the engine. The Foxboro dew-point measuring system using the Dewcel permits the continuous recording of the dew point to within $\pm 2^\circ$ F over a range of temperature from -20° to 120° F.

The humidity investigations were run in the following manner: The inlet temperature was set at a value sufficiently high to permit runs at several values of high specific humidity. The inlet pressure and temperature and exhaust pressure were held constant and engine speed was varied for two widely differing values of the specific humidity. For another portion of the experimental investigation, the corrected engine speed $N/\sqrt{\theta}$ was held constant and only the specific humidity was varied. The ram pressure ratio (flight Mach number) was maintained constant in both cases.

RESULTS AND DISCUSSION

Calculated Humidity Correction Factors

The humidity correction factors for the various engine performance parameters computed for the condition where the compressor Mach number remains constant with changing moisture conditions are given in figure 3. The curves labeled "Engine A, detailed method" are results obtained from the General Electric Company for a turbojet engine with a centrifugal-type compressor and a single-stage axial-flow turbine. The curves marked "Engine B, detailed method" are results obtained by the NACA using a different engine and the method of analysis presented in appendix B, a method similar to that used by the General Electric Company. A comparison of these two sets of results is made because the humidity corrections resulting from the detailed type of analysis involve engine parameters and a comparison of this nature affords an indication of the extent to which the humidity corrections will depend on the design of the engine. Although the two engines had similar type compressors and turbines, the rated thrust of engine B is approximately 185 percent higher and the compressor pressure ratio 16 percent higher than that of engine A.

The curves labeled "Approximate method" were obtained using the analysis developed herein, in which the thermodynamic properties were calculated for the operating temperatures of engine B.

All the performance parameters of the engine (fig. 3) vary linearly with increasing humidity. By the approximate method of analysis, the maximum change in the performance parameters for operation of the engine at constant compressor Mach number with varying humidity up to 300 grains of water per pound of dry air is: for engine speed, an increase of 1.0 percent; for air flow, a decrease of 1.4 percent; for fuel flow, an increase of 2.4 percent; for jet thrust, a decrease of 0.45 percent. The tail-pipe temperature (not shown for approximate method), of course, does not vary because the characteristic total-temperature ratio remains constant with varying humidity, as previously assumed. The performance of the engine at constant compressor Mach number with varying humidity is affected, in general, by less than 1.0 percent for all humidities of less than 140 grains of water per pound of dry air. Although the humidity correction factors for engine speed for the two methods of computation are the same, a small difference in the results for the two different engines is shown in figure 3(a). This difference is attributed to the use of slightly different values of the standard thermodynamic properties of dry air.

In view of the general magnitude of the correction factors, the agreement between the two methods of calculation seems to be sufficient to justify the use of the approximate method to determine humidity corrections to engine performance. The close agreement between the results of the detailed method as applied to two different engines seems to indicate that the humidity corrections determined for one engine could be applied to another engine of similar type.

Experimental Effect of Humidity on Engine Performance

Experimental results of the effect of humidity on engine performance are presented in figure 4(a). Although the effect of humidity is small and within the usual reproducibility of data, the data do indicate that there is some decrease in performance at the high humidity.

An inspection of the data of figure 4(a) indicates the effect of humidity at a given engine speed. If the compressor Mach number is maintained constant with varying humidity, however, the actual engine speed must be increased, as previously illustrated; therefore the effect of humidity at constant compressor Mach number is different from that shown in figure 4(a). For example, the variation of the jet thrust with humidity shown in figure 4(a) for a given engine speed is larger than that shown in figure 3(d) for a constant compressor Mach number.

The data presented in figure 4(b) are the same as those in figure 4(a) except that the humidity correction factors of the approximate method have been applied. The corrected data fall essentially on a single curve demonstrating the applicability of the correction factors.

Experimental results for which the performance of the engine was determined over a range of specific humidities at constant simulated altitude, ram pressure ratio, engine speed, and inlet temperature are presented in figure 5(a). The performance at a very low specific humidity (30 grains water/lb dry air) was considered as dry performance and the approximate humidity correction factors were applied in the inverse manner to determine the theoretical curves of figure 5(a). The operation of this particular engine with varying humidity (up to 210 grains water/lb dry air) produced a decrease of 2.0 percent in air flow and 3.6 percent in thrust. It appears possible that these values would increase to 2.5 and 4.5 percent, respectively, at a humidity of 300 grains of water per pound of dry air. It is shown in figure 5(a), as in figure 4(a), that the fuel flow will be little affected by changing humidity at this engine speed; however, at low speeds (fig. 4(a)) there is a greater humidity effect.

The theoretical curves (fig. 5(a)) were calculated by assuming a constant compressor Mach number to exist with a varying humidity, and the experimental curves resulted from operating the engine at constant engine speed and inlet temperature. The difference between the theoretical curves and the experimental curves must therefore be the result of the variation of engine speed with humidity for the theoretical curves. In order to substantiate this reasoning, figure 5(b) presents the theoretical curves adjusted to a constant engine speed in accordance with the effect of a variation in engine speed on performance similar to that in figure 4(b). Very good agreement exists between experiment and theory when the theoretical curves are corrected in this manner.

Variation of Saturation Specific Humidity with Altitude

In order to illustrate the humidites that may be encountered in the operation of turbojet engines, the variation of saturation specific humidity with altitude is presented in figure 6. It may be observed that the saturation specific humidity at low altitudes decreases rapidly with increasing altitude for both NACA standard atmosphere and Army summer air. By judging from the general magnitude of the humidity effect, no correction to performance for humidity need be made for an engine operating in NACA atmosphere or above 15,000 feet altitude in Army summer air. The humidity corrections under these conditions would most likely be less than 1 percent.

SUMMARY OF RESULTS

The following results were obtained from an investigation of the effect of humidity on turbojet-engine performance; two analytical methods were used in which the consequence of neglecting the variation of some of the fluid properties of air with humidity was determined:

Correction factors that are generally applicable to any engine and convenient to apply were developed to correct engine performance for humidity. These factors were verified by detailed performance analyses for two different engines and satisfactorily correlated experimental data obtained over a wide range of specific humidities.

The humidity effect on performance was small. Experimental results showed that thrust was affected most and that for a given engine speed this parameter decreased 3.6 percent for a variation in specific humidity from 30 to 210 grains of water per pound of dry air.

Lewis Flight Propulsion Laboratory,
National Advisory Committee for Aeronautics,
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APPENDIX A

SYMBOLS

The following symbols are used in this report: 1295

A	effective area, sq ft
a	speed of sound, ft/sec
C	constant
C_p	specific heat at constant pressure, Btu/(lb)(°R)
C_v	specific heat at constant volume, Btu/(lb)(°R)
F	jet thrust, lb
$f_1,f_2,f_3\ldots$	functions
f/a	fuel-air ratio
g	acceleration due to gravity, 32.2 ft/sec^2
ΔH	enthalpy change, Btu/lb
J	mechanical equivalent of heat, 778 ft-lb/Btu
K	constant, $(^{\circ}R)^{1/2}$/sec
L	characteristic length, ft
M	Mach number
N	engine speed, rpm
P	total pressure, lb/sq ft
p	static pressure, lb/sq ft
Q_L	lower heating value of fuel, Btu/lb
q	specific humidity, grains water vapor/lb dry air
R	gas constant, ft-lb/(lb)(°R)
S	slip factor of compressor

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T total temperature, °R

t static temperature, °R

V velocity, ft/sec

v velocity coefficient

W_a air flow, lb/sec

W_f fuel flow, lb/sec

W_g gas flow, lb/sec

γ ratio of specific heat at constant pressure to specific heat at constant volume

δ ratio of compressor-inlet total pressure to NACA standard sea-level pressure

η efficiency

θ ratio of compressor-inlet total temperature to NACA standard sea-level temperature

μ viscosity, lb/ft-sec

ρ static density, lb/cu ft

ψ compressor pressure coefficient

Subscripts:

0 ambient

1 compressor inlet

2 compressor discharge

3 turbine inlet

4 exhaust-nozzle throat

5 station at which jet is expanded to ambient pressure

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a	air (dry air plus water vapor)
b	combustion chamber
c	compressor
e	exhaust nozzle
m	moist air
n	dry air
st	standard
t	turbine
w	water vapor

APPENDIX B

DETAILED METHOD OF DETERMINING ENGINE PERFORMANCE

WITH VARYING HUMIDITY

In general, for an engine of fixed configuration, maintenance of a constant flight Mach number and a constant compressor Mach number with varying humidity will not insure that the turbine and exhaust-nozzle Mach numbers will remain constant. Determination of the complete performance of the engine by a cycle analysis is therefore necessary in order to obtain exactly the change in the generalized engine performance parameters due to humidity.

The performance of a turbojet engine may be related at two moisture conditions under the following restrictions:

(1) All engine efficiencies, effective flow areas, and flow coefficients remain constant with varying humidity.

(2) The compressor Mach number, the inlet total temperature and pressure, and the exhaust static pressure remain constant with varying humidity. (The condition of constant compressor Mach number imposes a humidity engine-speed correction.)

(3) Sonic velocity exists in the turbine-stator throat.

Let the dry and wet conditions be represented by subscripts n and m, respectively. In order to maintain a constant compressor Mach number, based on inlet total temperature, with varying humidity (assuming that the velocity of the air entering the compressor is low and that total and static temperature are equal), the engine speed should be altered by the following ratio:

$$\frac{N_n}{N_m} = \sqrt{\frac{\gamma_{c,n} R_{c,n}}{\gamma_{c,m} R_{c,m}}} \tag{B1}$$

The ratio of the compressor-discharge pressures at the two moisture conditions is

$$\frac{P_{2,n}}{P_{2,m}} = \frac{\left(1 + \frac{\psi V_{c,n}^2}{gJc_{p,c,n}T_{1,n}}\right)^{\frac{\gamma_{c,n}}{\gamma_{c,n}-1}}}{\left(1 + \frac{\psi V_{c,m}^2}{gJc_{p,c,m}T_{1,m}}\right)^{\frac{\gamma_{c,m}}{\gamma_{c,m}-1}}} \tag{B2}$$

The ratio of the compressor-discharge temperatures at the two moisture conditions is

$$\frac{T_{2,n}}{T_{2,m}} = \frac{1 + \dfrac{s\, V_{c,n}^{2}}{gJC_{p,c,n}T_{1,n}}}{1 + \dfrac{s\, V_{c,m}^{2}}{gJC_{p,c,m}T_{1,m}}} \tag{B3}$$

The assumption of a choked stator permits the expression

$$\frac{W_{g,3}\sqrt{T_3}}{A_3P_3} = \sqrt{\frac{\gamma_t g}{R_t}\left(\frac{2}{\gamma_t+1}\right)^{\frac{\gamma_t+1}{\gamma_t-1}}} = K \tag{B4}$$

The weight-flow parameter at station 3 may be rewritten

$$\frac{W_{g,3}\sqrt{T_3}}{A_3P_3} = \left(\frac{W_{g,4}\sqrt{T_4}}{A_4P_4}\right)\left(\frac{P_4}{P_3}\right)\left(\frac{T_3}{T_4}\right)^{1/2}\left(\frac{A_4}{A_3}\right) = K \tag{B5}$$

When the weight-flow parameter at station 4 is expanded in terms of pressure ratios, equation (5A) becomes

$$\frac{W_{g,3}\sqrt{T_3}}{A_3P_3} = \sqrt{\frac{2g\gamma_e}{R_e(\gamma_e-1)}}\left[\left(\frac{P_4}{P_3}\right)^{\frac{2}{\gamma_e}}\left(\frac{P_3}{P_4}\right)^{\frac{2}{\gamma_e}} - \left(\frac{P_4}{P_3}\right)^{\frac{\gamma_e+1}{\gamma_e}}\left(\frac{P_3}{P_4}\right)^{\frac{\gamma_e+1}{\gamma_e}}\right]^{1/2}\frac{P_4}{P_3}\left(\frac{T_3}{T_4}\right)^{1/2}\frac{A_4}{A_3} = K \tag{B6}$$

The turbine temperature ratio is related to the turbine pressure ratio by

$$\frac{T_3}{T_4} = \left\{1 - \eta_t \left[1 - \left(\frac{P_4}{P_3}\right)^{\frac{\gamma_t - 1}{\gamma_t}}\right]\right\}^{-1} \tag{B7}$$

Equations (B6), (B7), and (B2) may be used to obtain the turbine pressure ratio if p_4 is taken equal to p_0 and if the burner pressure drop is assumed constant so that P_3 equals CP_2. The assumption of ambient static pressure in the exhaust-nozzle throat restricts this part of the analysis to applications where the nozzle is not choked; however, it will be shown later that the resulting factors may be determined for all operating conditions by the introduction of an additional assumption.

When the turbine pressure ratio is determined, it is possible to determine the turbine-inlet temperature. Equating the actual work of the compressor and the turbine gives

$$\frac{T_3}{T_1} = \frac{\dfrac{SV_c^{\,2}}{gJT_1}}{\eta_t c_{p,t}\left[1 - \left(\frac{P_4}{P_3}\right)^{\frac{\gamma_t - 1}{\gamma_t}}\right](1+f/a)} \tag{B8}$$

The humidity correction factor for turbine-inlet temperature is therefore

$$\frac{T_{3,n}}{T_{3,m}} = \frac{V_{c,n}^{\,2}}{V_{c,m}^{\,2}}\,\frac{c_{p,t,m}}{c_{p,t,n}}\;\frac{\left[1 - \left(\frac{P_4}{P_3}\right)_m^{\frac{\gamma_{t,m} - 1}{\gamma_{t,m}}}\right]\left[1+(f/a)_m\right]}{\left[1 - \left(\frac{P_4}{P_3}\right)_n^{\frac{\gamma_{t,n} - 1}{\gamma_{t,n}}}\right]\left[1+(f/a)_n\right]} \tag{B9}$$

Equation (B9) can be used only for an unchoked jet nozzle because P_3/P_4 cannot be evaluated in equation (B6) for the choked condition.

An approximation to the humidity correction factors, however, may be obtained if the turbine-inlet temperature is assumed constant with varying humidity as it is in the analysis in the text. It is believed this approximation introduces negligible error in the correction factors.

Knowledge of the turbine-inlet temperature permits easy calculation of the other engine variables. The fuel-air ratio may be obtained from the following development:

$$\eta_b = \frac{W_g \Delta H_b}{W_f \; Q_L} \tag{B10}$$

Therefore

$$f/a = \frac{\Delta H_b}{\eta_b Q_L - \Delta H_b} \tag{B11}$$

Assuming a value for the combustion efficiency, the fuel-air ratio humidity correction factor is

$$\frac{(f/a)_n}{(f/a)_m} = \frac{\Delta H_{b,n}}{\Delta H_{b,m}} \left(\frac{\eta_b Q_L - \Delta H_{b,m}}{\eta_b Q_L - \Delta H_{b,n}} \right) \tag{B12}$$

The weight flow through the turbine stator is

$$W_{g,3} = K \frac{P_3 A_3}{\sqrt{T_3}} \tag{B13}$$

and the humidity correction factor is

$$\frac{W_{g,3,n}}{W_{g,3,m}} = \frac{K_n}{K_m} \sqrt{\frac{T_{3,m}}{T_{3,n}}} \frac{P_{3,n}}{P_{3,m}} \tag{B14}$$

The air-flow humidity correction is

$$\frac{W_{a,n}}{W_{a,m}} = \frac{K_n}{K_m} \sqrt{\frac{T_{3,m}}{T_{3,n}}} \frac{P_{3,n}}{P_{3,m}} \frac{\left[1 + (f/a)_m\right]}{\left[1 + (f/a)_n\right]} \tag{B15}$$

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and the fuel-flow humidity correction factor is

$$\frac{W_{f,n}}{W_{f,m}} = \frac{K_n}{K_m}\sqrt{\frac{T_{3,m}}{T_{3,n}}}\,\frac{P_{3,n}}{P_{3,m}}\,\frac{\Delta H_{b,n}}{\Delta H_{b,m}} \tag{B16}$$

The jet-nozzle total temperature may be computed from equations (B7) and (B8); the effective jet velocity is then obtained from

$$V_5 = v\sqrt{2gJC_{p,e}T_4\left[1-\left(\frac{p_0}{P_4}\right)^{\frac{\gamma_e-1}{\gamma_e}}\right]} \tag{B17}$$

The jet-velocity humidity correction is

$$\frac{V_{5,n}}{V_{5,m}} = \sqrt{\frac{C_{p,e,n}}{C_{p,e,m}}}\sqrt{\frac{T_{4,n}}{T_{4,m}}}\sqrt{\frac{1-\left(\frac{p_0}{P_4}\right)_n^{\frac{\gamma_{e,n}-1}{\gamma_{e,n}}}}{1-\left(\frac{p_0}{P_4}\right)_m^{\frac{\gamma_{e,m}-1}{\gamma_{e,m}}}}} \tag{B18}$$

where

$$\frac{T_{4,n}}{T_{4,m}} = \frac{T_{3,n}}{T_{3,m}}\,\frac{\left\{1-\eta_t\left[1-\left(\frac{P_4}{P_3}\right)_n^{\frac{\gamma_{t,n}-1}{\gamma_{t,n}}}\right]\right\}}{\left\{1-\eta_t\left[1-\left(\frac{P_4}{P_3}\right)_m^{\frac{\gamma_{t,m}-1}{\gamma_{t,m}}}\right]\right\}} \tag{B19}$$

and is the tail-pipe temperature humidity correction factor. Because the jet velocity and the weight flow are known, the jet thrust is readily obtained from

$$F = \frac{W_{g,4}V_5}{g} = \frac{W_{g,3}V_5}{g} \qquad \text{(B20)}$$

and the jet-thrust humidity correction factor is

$$\frac{F_n}{F_m} = \frac{W_{g,3,n}}{W_{g,3,m}} \frac{V_{5,n}}{V_{5,m}} \qquad \text{(B21)}$$

In the preceding analysis, the performance of the engine is related at two moisture conditions similar to the method used by the General Electric Company. In order to study the variation of performance due to humidity for a given compressor Mach number and flight condition, only insertion of the proper thermodynamic constants into the equations of performance is necessary.

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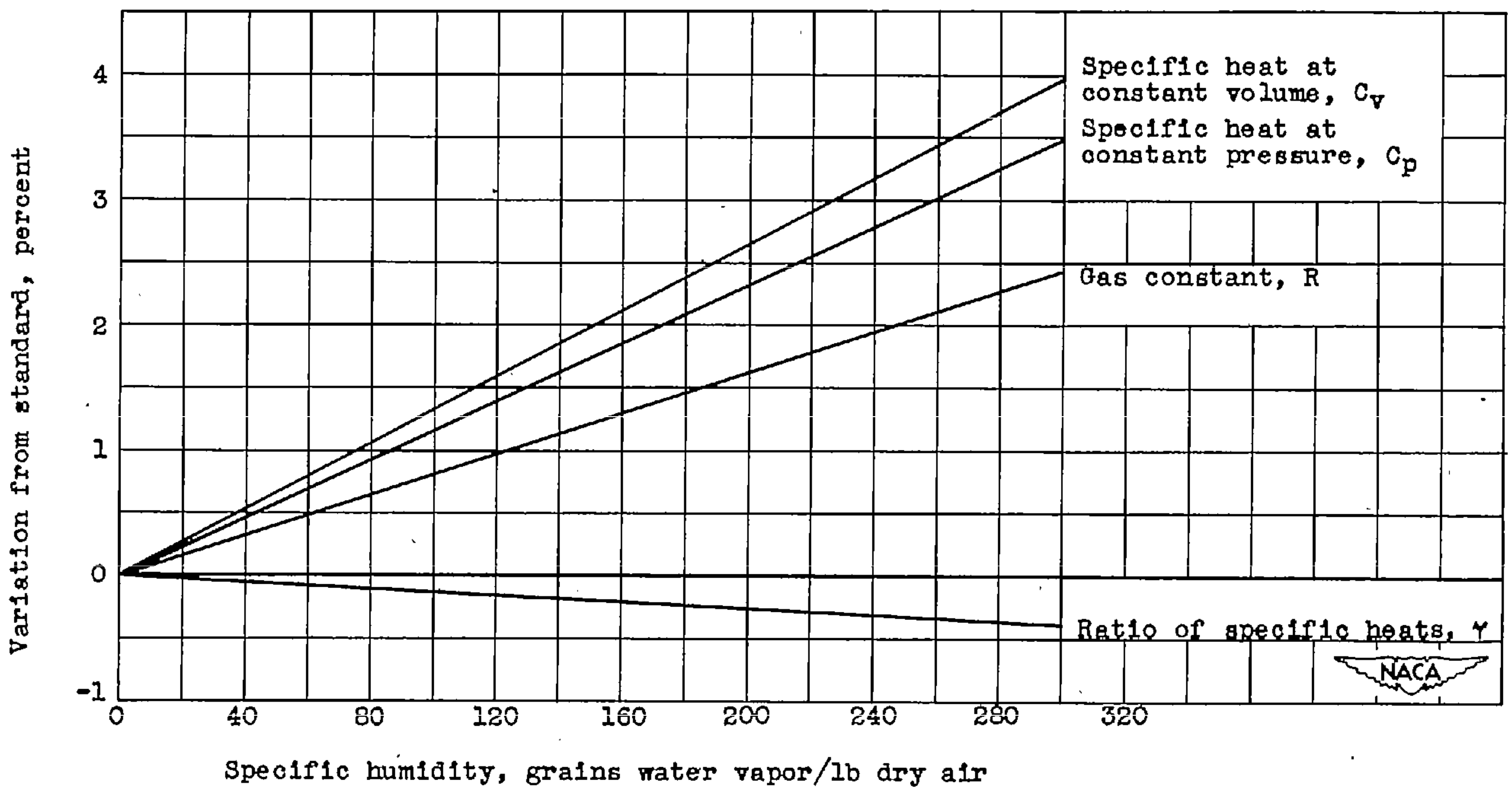


Figure 1. - Exemplary variation of thermodynamic properties of moist air as function of specific humidity. Standard values of constants taken at 59° F: C_p, 0.2397; C_v, 0.1711; γ, 1.401; R, 53.39 for dry air.

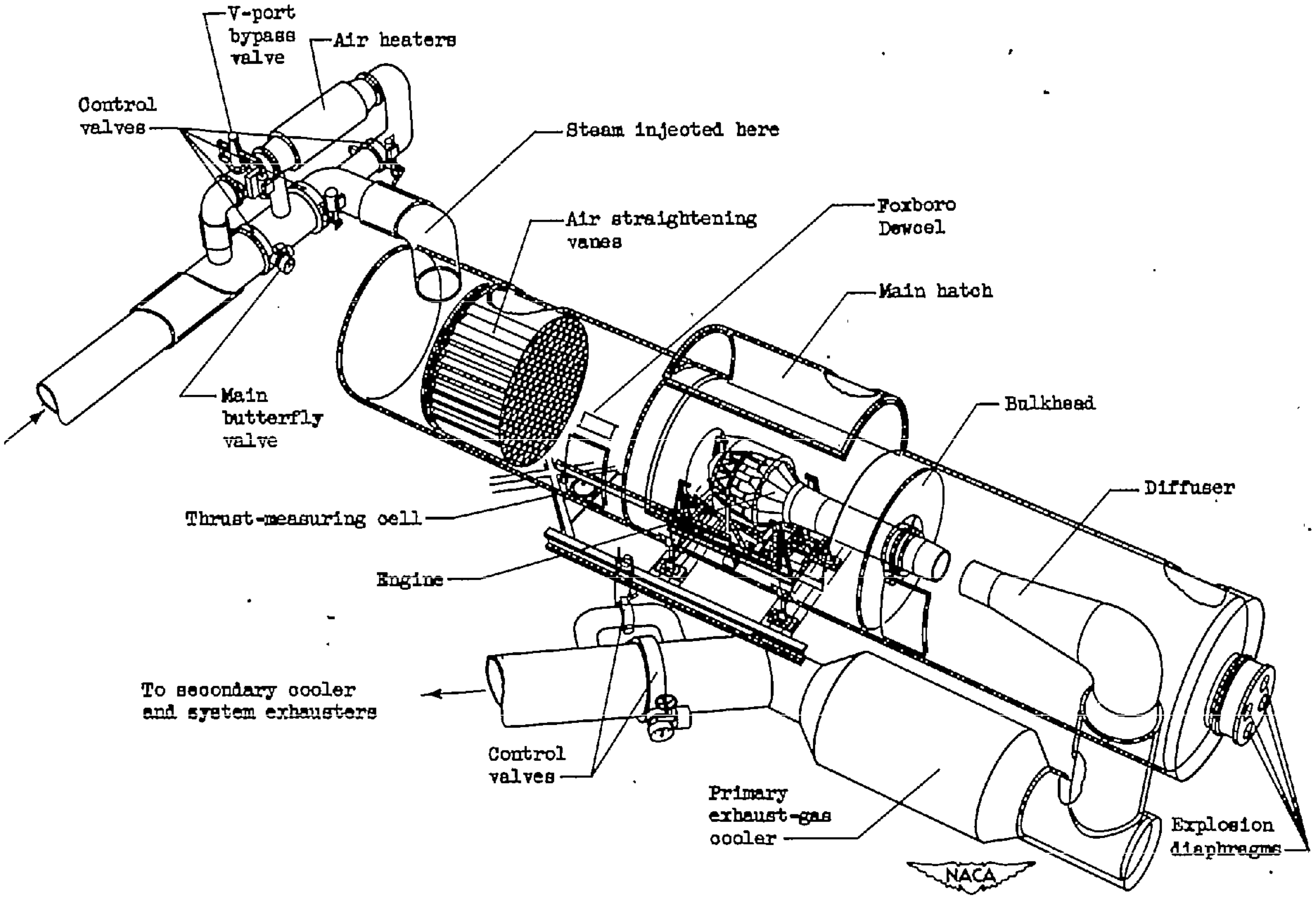
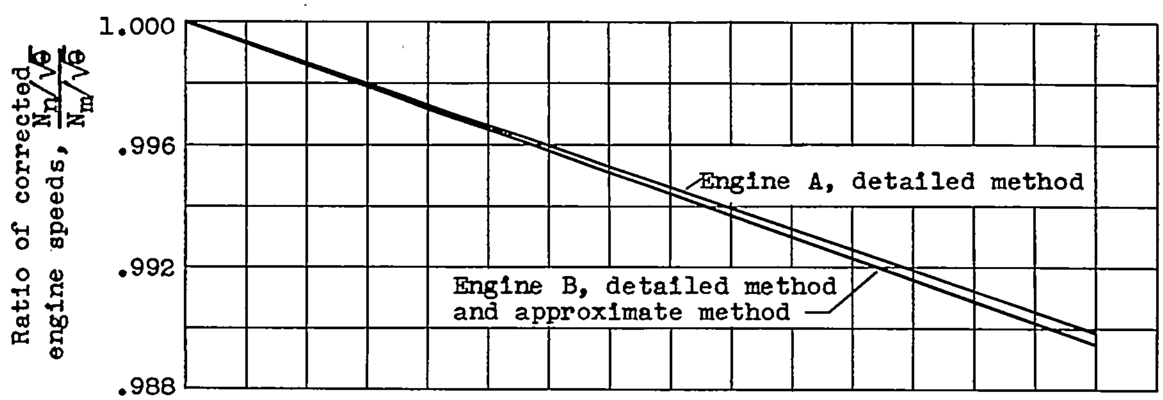
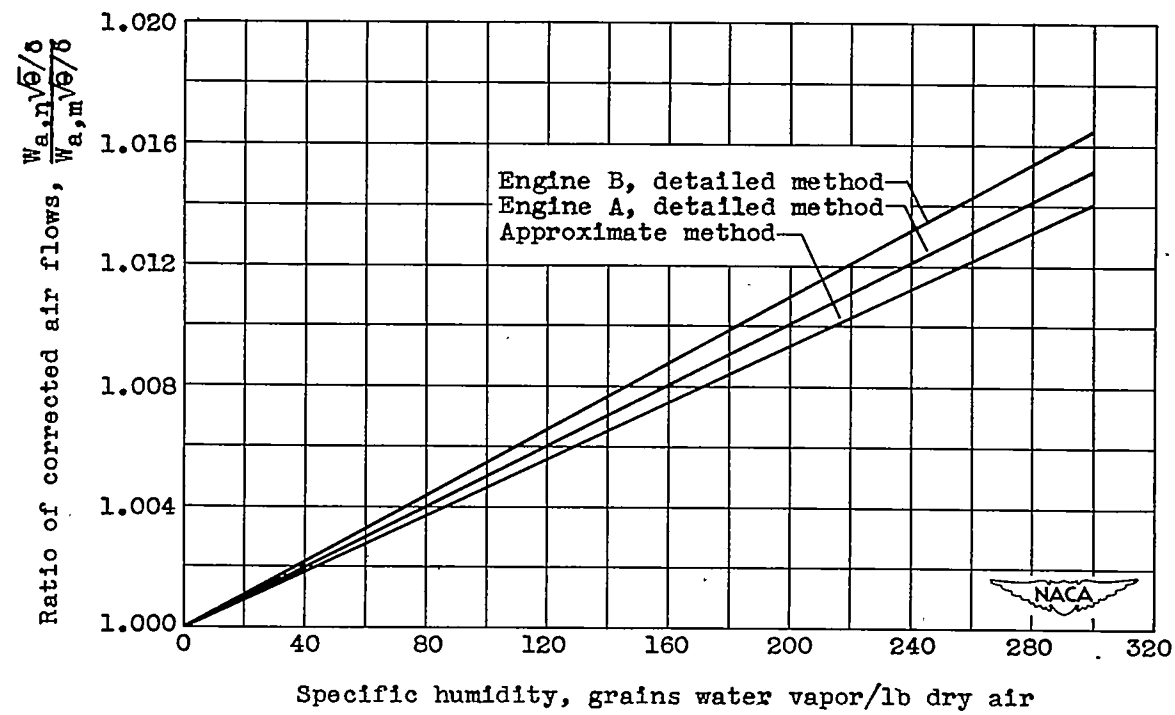


Figure 2. - Altitude chamber with engine installed in test section.

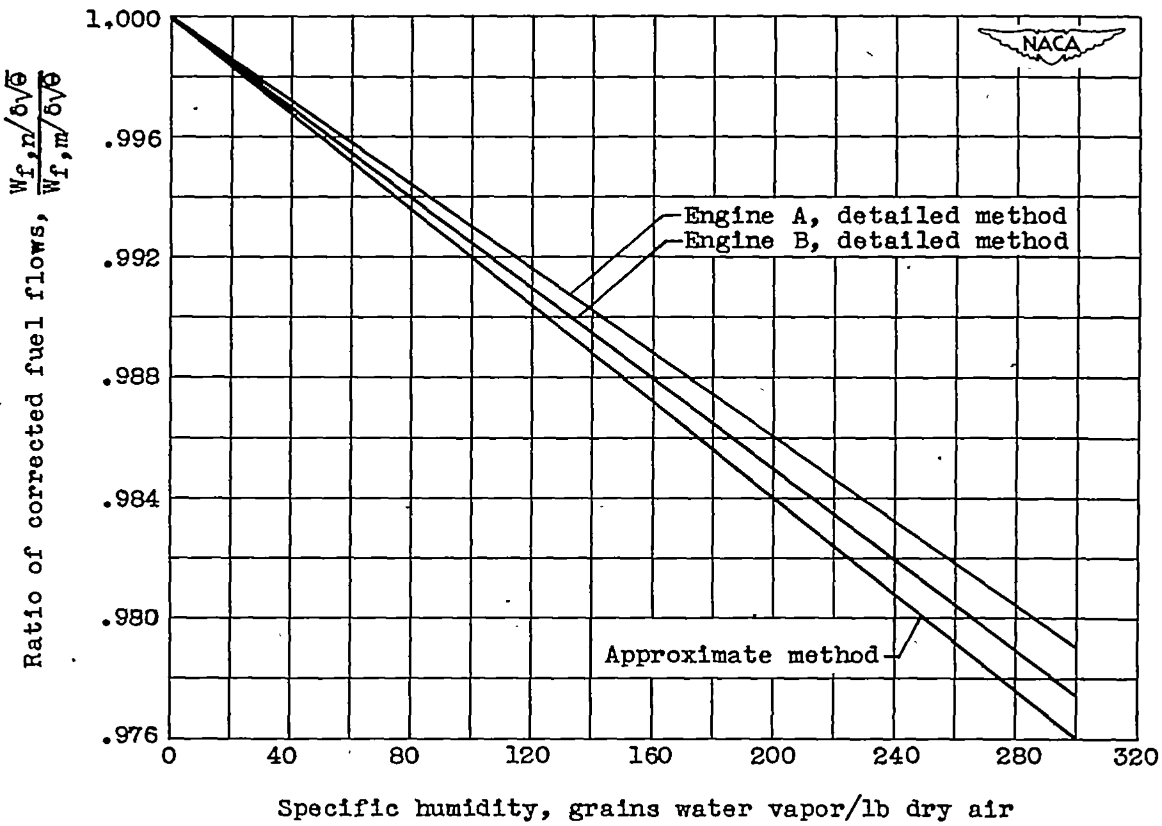


(a) Engine speed.



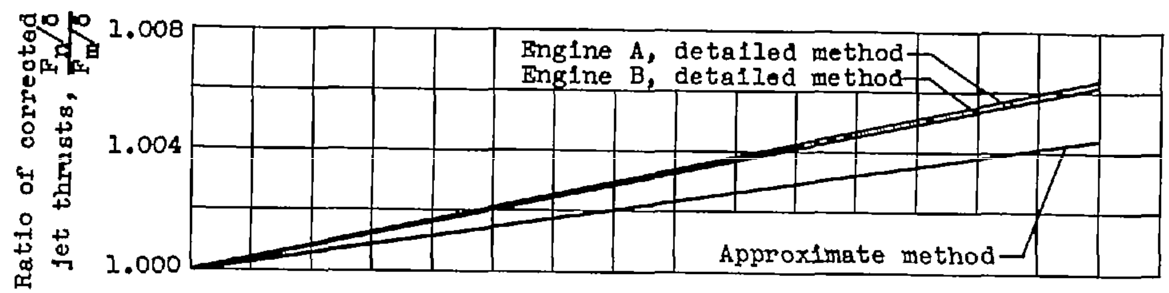
(b) Air flow.

Figure 3. - Correction for humidity at constant compressor Mach number. (Based on maximum engine speed, sea-level static-pressure operation.)

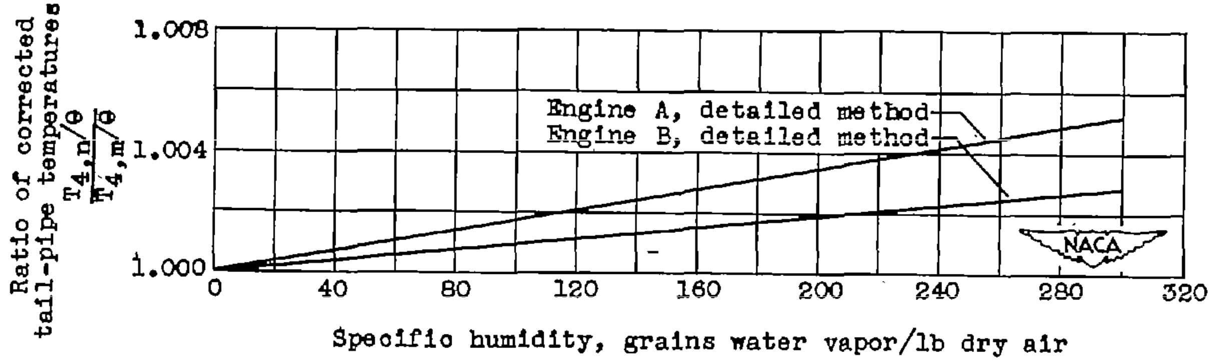


(c) Fuel flow.

Figure 3. - Continued. Correction for humidity at constant compressor Mach number. (Based on maximum engine speed, sea-level static-pressure operation.)

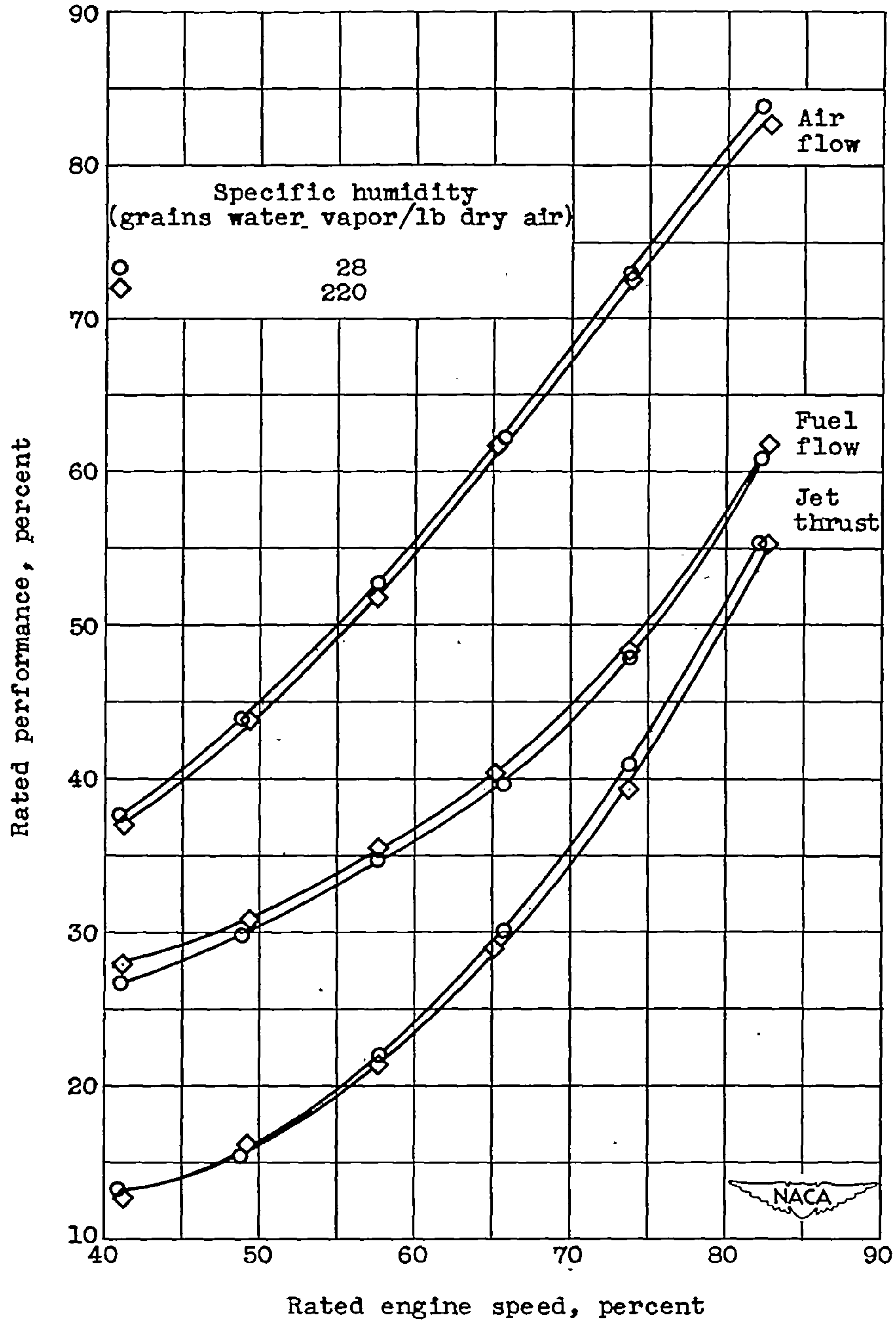


(d) Jet thrust.



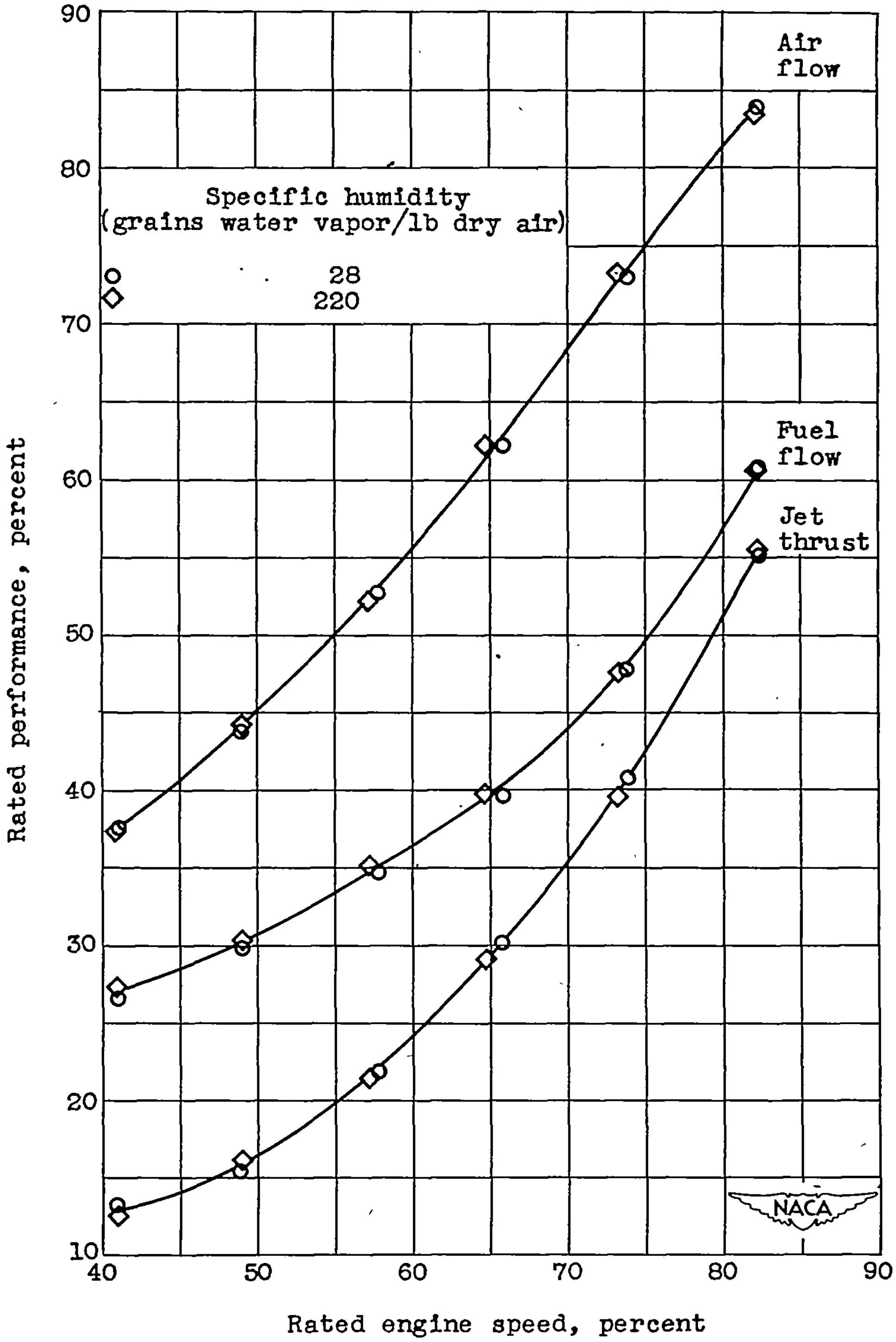
(e) Tail-pipe temperature.

Figure 3. - Concluded. Correction for humidity at constant compressor Mach number. (Based on maximum engine speed, sea-level static-pressure operation.)



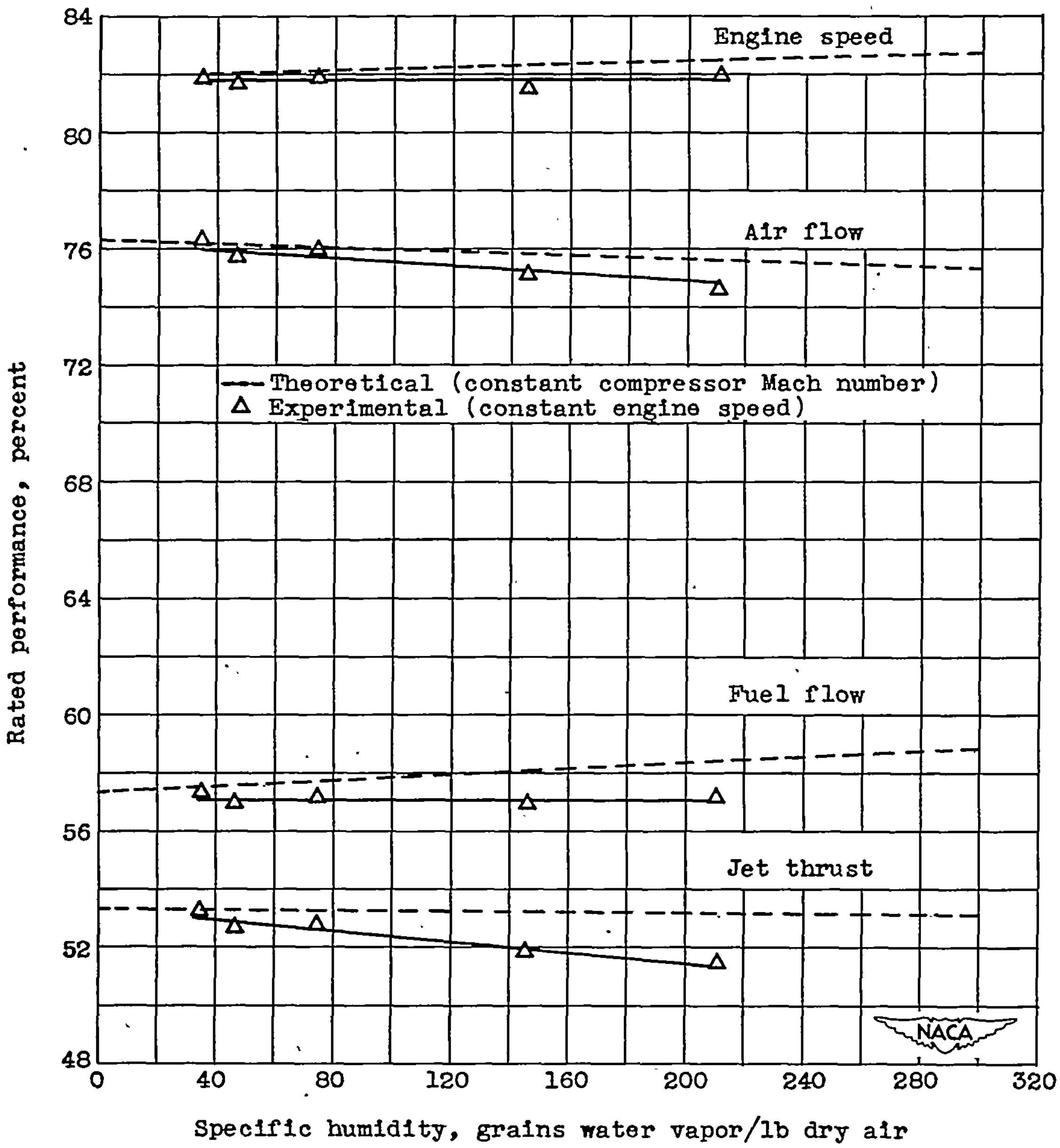
(a) Data uncorrected for humidity.

Figure 4. - Effect of humidity on engine performance. Sea-level static pressure; inlet temperature, 93° F.



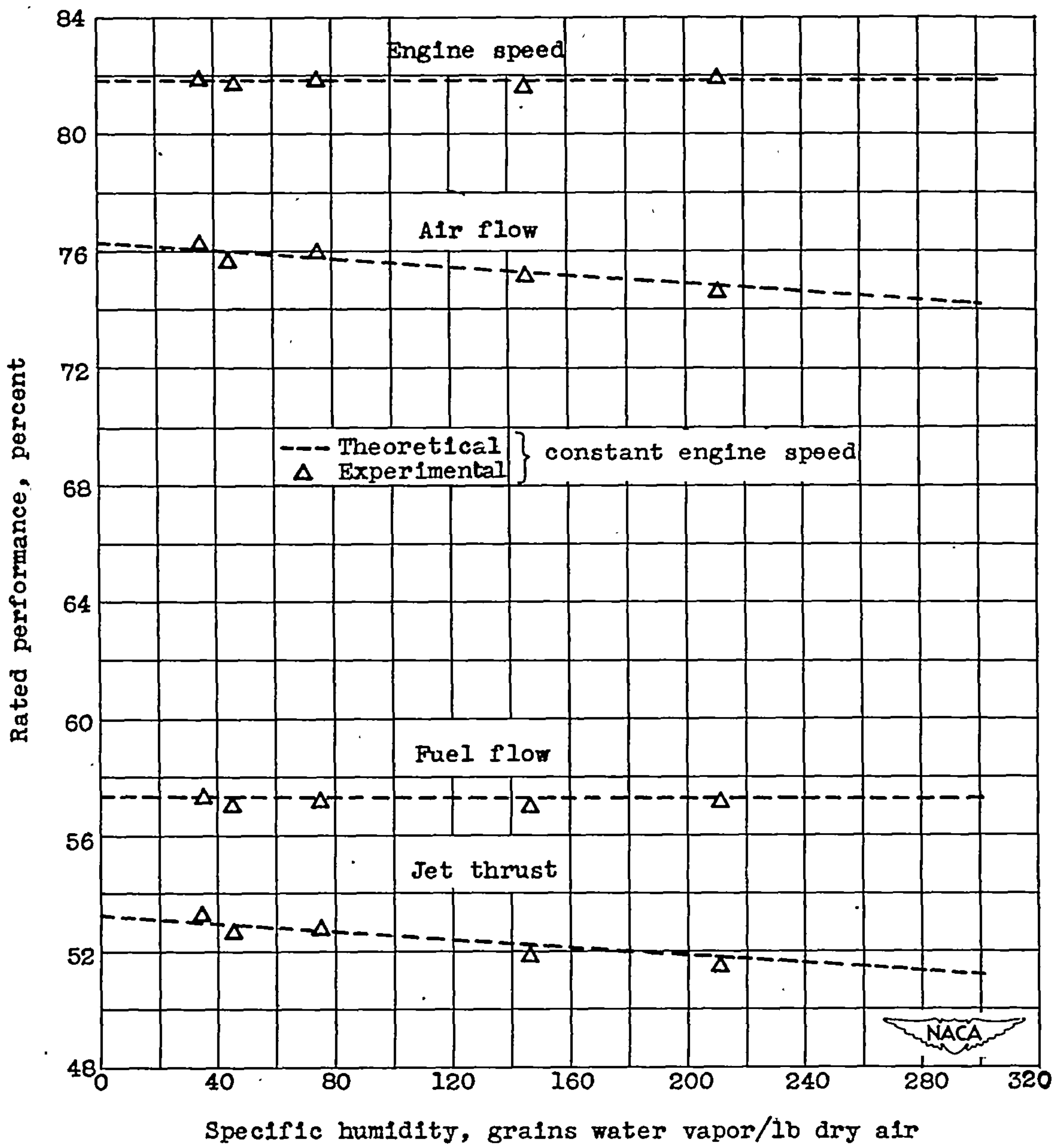
(b) Data corrected for humidity.

Figure 4. - Concluded. Effect of humidity on engine performance. Sea-level static pressure; inlet temperature, 93° F.



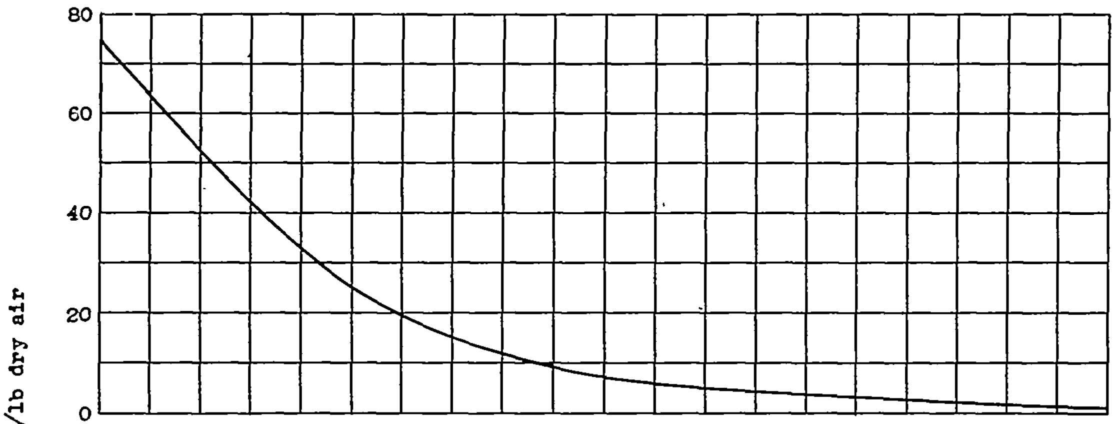
(a) Theoretical curves are at constant compressor Mach number; experimental data are at constant engine speed.

Figure 5. - Effect of humidity on engine performance. Sea-level ram pressure ratio, 1.1; inlet temperature, 110° F.

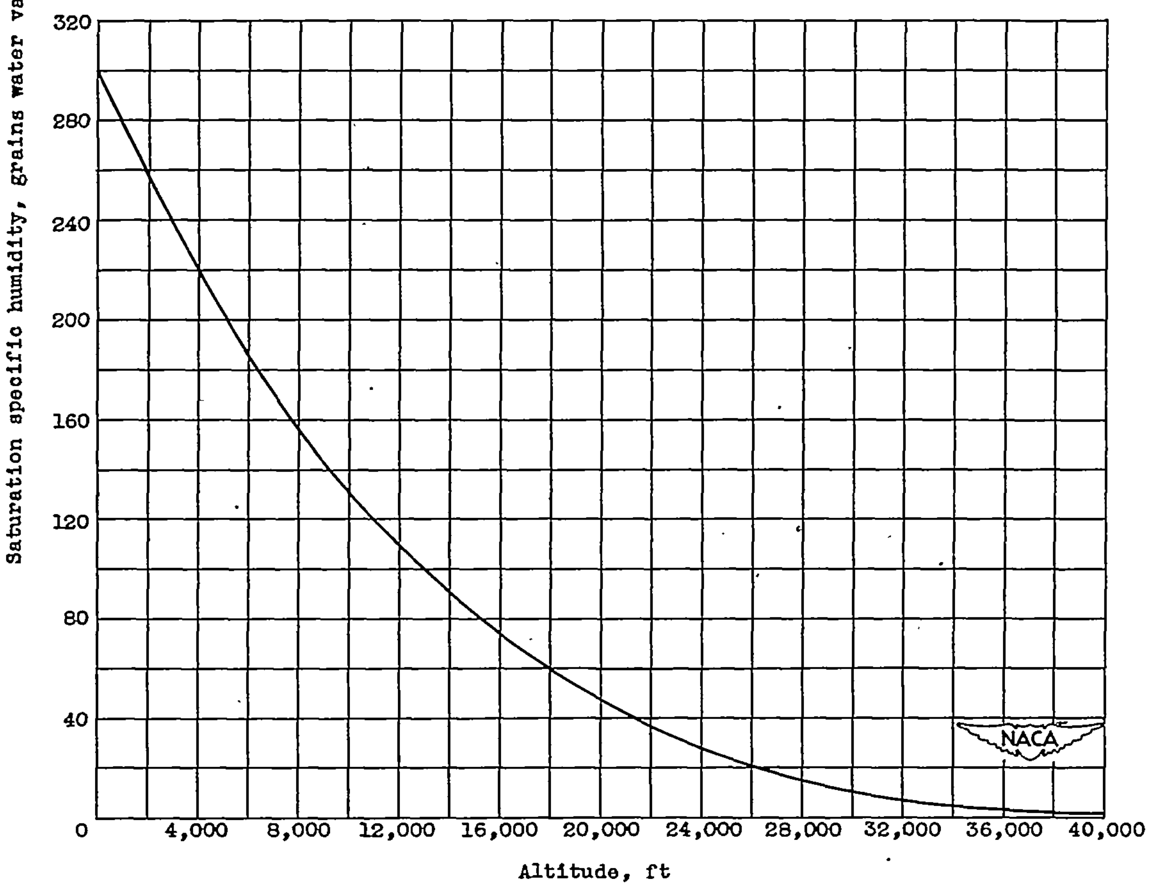


(b) Theoretical curves and experimental data are at constant engine speed.

Figure 5. - Concluded. Effect of humidity on engine performance. Sea-level, ram pressure ratio, 1.1; inlet temperature, 110° F.



(a) NACA standard atmosphere.



(b) Army summer air.

Figure 6. - Variation of saturation specific humidity with altitude.

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